Transformation Matrix Algorithm for Reducing the Computational Complexity of Multiuser Detectors for DS-CDMA Systems

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Abstract

Multiuser detection is an important technology in wireless CDMA systems for improving both data rate as well as user capacity. However, the computational complexity of multiuser detection prevents the widespread use of this technique. Most of the CDMA systems today and in the near future will continue to use the conventional matched filter with its comparatively low user capacity and a slow data rate. However, if we could lower the computational complexity of multiuser detectors, most of the CDMA systems would be likely to take advantage of this technique in order to gain increased system capacity and a better data rate. In this paper, a novel approach for reducing the computational complexity of multiuser receivers is proposed. It utilizes the transformation matrix technique to improve the performance of multiuser detectors. We show that the mathematical computations of the implementation complexity can turn in overall less complex system that has strong impact on the system's signal to noise ratio (SNR) and the bit error rate (BER). The performance measure adopted in this paper is the achievable bit rate for a fixed probability of error (10^{-7}) and consistent values of SNR

1. Introduction

Multiuser direct-sequence code division multiple access (DS-CDMA) has received wide attention in the field of wireless communications [1, 2]. With the emergence of multiple access techniques, there has been an increase in the interest in performing simultaneous estimation and detection over all users [4]. Multiple access interference (MAI) can be prevented by selecting mutually orthogonal signature waveforms for all the active users. However, it is not possible to ensure perfect orthogonality among received signature waveforms in a mobile environment, and thus MAI arises. Multiuser detection is a technique to improve the capacity and coverage in a CDMA system. Being a critical component of this technique, the maximum likelihood (ML) multiuser receiver has received extensive study. However, the computational complexity of this receiver prevents the widespread use of this technique. Due to the high computational complexity, most of the CDMA systems today and in the near future will continue to use the conventional matched filter with comparatively low user capacity and a slow data rate.

While many multiuser detectors could achieve optimal performance, their relatively higher complexity prevents CDMA systems from adapting this technology for signal detection. However, if we could lower the computational complexity of multiuser detectors, most CDMA systems would likely take advantage of this technique in order to gain increased system capacity and a better data rate. In this paper, a novel approach for reducing the asymptotic computational complexity of multiuser receivers is proposed that utilizes the transformation matrix technique to improve the performance of multiuser detectors. By using the proposed algorithm, the computational complexity of multiuser detectors can be reduced by several orders of magnitude. This is done by realizing that much of the processing performed is unnecessary. Since most of the decisions are correct, we can reduce the number of computations by using the transformation matrices only on those coordinates that are most likely to lead to an incorrect decision. By doing this, we can greatly reduce the processing that was required to make a decision about the correct region or the coordinate. Thus, this reduction in the computational complexity will likely give us a considerable improvement in the performance of multiuser receivers. The performance measure adopted in this paper is the achievable bit rate for a fixed probability of error (10^{-7}) and consistent values of SNR.

Verdu [3] proposed and analyzed the optimum ML sequence detector which, unfortunately, is too complex for practical implementation, since its complexity grows exponentially as a function of the number of users. Consequently, Verdu's work has inspired researchers to search for suboptimal multiusers detectors, which can achieve near-optimal performance with comparatively less computational complexity. Suboptimal receivers also simultaneously detect all user signals. However, instead of ML detection, they perform a set of linear transformations on the outputs of a matched filter that significantly enhances the noise component.

The rest of this paper is organized as follows: Section 2 describes the research that has already been done in this area. Section 3 presents the ML algorithm and the proposed algorithm, with section 3.1 covering the ML algorithm and its corresponding computational complexity and section 3.2 covering the proposed transformation matrix algorithm. The mathematical derivations for generating consistent values of SNR and the standard formulas for BER are presented in sections 3.3 and 3.4, respectively. The mathematical and simulation results of SNR and BER performance are provided in section 4. Finally, section 5 concludes the paper.

2. Related work

Multiuser receivers can be categorized in the following two forms: optimal ML sequence estimation (MLSE) receivers and suboptimal linear and nonlinear receivers. It has been shown in [9] that DS-CDMA is not fundamentally MAI limited and can be near-far resistant. Optimal multiuser detection consists of a matched filter followed by a ML sequence detector implemented via a dynamic parallel programming algorithm. In order to mitigate the problem of MAI, Verdu [9] proposed and analyzed the optimum multiuser detector for asynchronous Gaussian multiple access channels. The optimum ML receiver searches all the possible demodulated bits in order to find the decision region that maximizes the correlation metric given by [3]. The practical application of this mechanism is limited by the complexity of the receiver [10]. This optimal detector outperforms the conventional detector, but unfortunately its complexity grows exponentially with a complexity of $O\,(2)^{K}$, where K is the number of active users.

Much research has been done to reduce this receiver's computational complexity. Recently, Ottosson and Agrell [8] proposed a ML receiver that uses the neighboring decent (ND) algorithm. They implemented an iterative approach using the ND algorithm to locate the region where the actual observations belong. In order to reduce the computational complexity of optimum receivers, the iterative approach using the ND algorithm performs MAI cancellation linearly. The linearity of the iterative approach increases noise components at the receiving end. Due to the enhancement in the noise components, the SNR and BER of the ND algorithm is more affected by the MAI.

Several tree-search detection receivers have been proposed in the literature [13, 14], in order to reduce the complexity of the original ML detection scheme proposed by Verdu. Specifically, [13] investigated a tree-search detection algorithm, where a recursive, additive metric was developed in order to reduce the search complexity. Reduced tree-search algorithms, such as the well known M-algorithms and T-algorithms were used by [14] in order to reduce the complexity incurred by the optimum multiuser detectors. In addition, an optimal MMSE receiver requires the inversion of a large matrix. This computation takes a relatively long time and makes the detection process slow and expensive [10, 11]. Xie, Rushforth, Short and Moon [15] proposed an approximate MLSE solution known as the pre-survivor processing (PSP) type algorithm, which combined a tree search algorithm for data detection with the aid of the recursive least square (RLS) adaptive algorithm used for channel amplitude and phase estimation. MLSE receivers give optimum performance but at a cost of increased receiver computational complexity.

In this paper, we employ a new approach using a transformation matrix algorithm that observes the coordinates of the constellation diagram to determine the location of the transformation points. Since most of the decisions are correct, we can reduce the number of required computations by using transformation matrices only on those coordinates which are most likely to lead to an incorrect decision. By doing this, we can greatly reduce the unnecessary processing involved in making decisions about the correct region or coordinates. Our mathematical results show that the proposed approach successfully reduces the computational complexity of the optimal ML receiver. The complexity of the proposed algorithm is not polynomial with respect to the number of users, but it still gives a comparatively reduced complexity and provides much better performance in terms of SNR and the BER than other well known multiuser detector algorithms such as ML and ND. The simulation results of the proposed algorithm demonstrate the positive impact of the reduced computational complexity by means of consistent values of SNR and the optimal BER performance.

3. The computational complexity of multiuser receivers

We consider a synchronous DS-CDMA system as a linear time invariant (LTI) channel. In an LTI channel, the probability of variations in the interference parameters, such as the timing of all users is extremely low. Our transformation matrix algorithm utilizes the complex properties of the existing inverse matrix algorithms to construct the transformation matrices and to determine the location of the transformation points that may occur in any coordinates of the constellation diagram. The individual transformation points can be used to determine the average computational complexity. The system may consist of K users. User k can transmit a signal at any given time with the power of W_k. With the binary phase shift keying (BPSK) modulation technique, the transmitted bits belong to either +1 or -1, i.e., $b_k \in \{\pm 1\}$.

3.1. The computational complexity of The ML algorithm

In order to mitigate the problem of MAI, Verdu [3] proposed and analyzed the optimum multiuser detector for

Gaussian multiple access channels. When a receiver wants to detect the signal from user-1, it first demodulates the received signal to obtain the base-band signal. The base-band signal is multiplied with user-1's unique signature waveform, $C_1(t)$. The resulting signal, $r_1(t)$, is applied to the input of the matched filter. The outputs of the matched filter and Verdu's algorithm can be represented by $y_k(m)$ and $b_k(m)$, respectively where m is the sampling interval. The outputs of the matched filter for the first two users at the m^{th} sampling interval can be expressed as follows:

$$y_1(m) = \frac{1}{T}\left\{\int_{(m)T}^{(m+1)T} r_1(t)C_1(t)dt\right\} \quad (1)$$

$$y_2(m) = \frac{1}{T}\left\{\int_{2+(m)T}^{2+(m+1)T} r_2(t)C_2(t-\tau_2)dt\right\} \quad (2)$$

The received signal $r_1(t)$ and $r_2(t)$ can be expressed as follows:

$$r_1(t) = \left(E_{C_1}\right)^{0.5}\sum_{i=-M}^{M} b_1(i)C_1(t-iT_b) \quad (3)$$

$$r_2(t) = \left(E_{C_2}\right)^{0.5}\sum_{i=-M}^{M} b_2(i)C_2(t-iT_b-\tau_2) \quad (4)$$

where E_{C_1} and E_{C_2} represent the original bit energy of the received signals. The received signals $r_1(t)$ and $r_2(t)$ can be treated as a single signal $r(t)$ that will be distinguished by the receiver with respect to its unique signature waveform. Substitute (3) and (4) as an individual equation into (1) and (2), respectively, and we get

$$y_1(m)=\frac{1}{T}\left\{\int_{(m)T}^{(m+1)T}\left(E_{C_1}\right)^{0.5}\left\{\sum_{i=-M}^{M}\left\{b_1(i)C_1\left(t-iT_b\right)\right\}\right\}C_1(t)dt\right\} \quad (5)$$

$$y_2(m)=\frac{1}{T}\left\{\int_{(m)T}^{(m+1)T}\left(E_{C_2}\right)^{0.5}\left\{\sum_{i=-M}^{M}\left\{b_2(i)C_2\left(t-iT_b-\tau_2\right)\right\}\right\}C_2\left(t-\tau_2\right)dt\right\} \quad (6)$$

After performing integration over the given interval, we get the following results with the noise components as well as the cross correlation of signature waveforms.

$$y_1(m) = \left(E_{C_1}\right)^{0.5} b_1(m) + \left(E_{C_2}\right)^{0.5} b_2(m-1)\rho_1 + \left(E_{C_2}\right)^{0.5} b_2(m)\rho_0 + \left(E_{C_2}\right)^{0.5} b_2(m+1)\rho_{-1} + n_1(m) \quad (7)$$

$$y_2(m) = \left(E_{C_2}\right)^{0.5} b_2(m) + \left(E_{C_1}\right)^{0.5} b_1(m-1)\rho_1 + \left(E_{C_1}\right)^{0.5} b_1(m)\rho_0 + \left(E_{C_1}\right)^{0.5} b_1(m+1)\rho_{-1} + n_2(m) \quad (8)$$

where coefficients $b_1(m)$ and $b_2(m)$ represent MAI, $\rho_{-1/0/+1}$ are cross-correlations of signature waveforms, and $n_1(m)$ and $n_2(m)$ represent the minimum noise components. These symbols can now be decoded using a ML Viterbi decision algorithm. This algorithm makes a decision over a finite window of sampling instants rather than waiting for all the data to be received [5]. The above derivation can be extended from two users to K users. The number of operations performed in the Viterbi algorithm is proportional to the number of decision states, and the number of decision states is exponential with respect to the total number of users. In other words, the computational complexity grows exponentially with respect to the total number of users. The computational complexity of this algorithm can be approximated as: $\boldsymbol{O}(2)^k$.

3.2. The computational complexity of the proposed algorithm

According to original Verdu's algorithm, the outputs of the matched filter $y_1(m)$, *and* $y_2(m)$ can be considered as a single output $y(m)$. In order to minimize the noise components and to maximize the received demodulated bits, we can transform the output of the matched filter, and this transformation can be expressed as follows: $y(m) = \mathrm{T}b + \eta$ where T represents the transformation matrix, $b_k \in \{\pm 1\}$ and η represents the noise components. In addition, if the vectors are regarded as points in K-dimensional space, then the vectors constitute a constellation diagram that has K total points. This constellation diagram can be mathematically expressed as: $\aleph = \{\mathrm{T}b\}$ *where* $b \in \{-1, +1\}$. The preceding equation is fundamental to the proposed algorithm. According to the detection rule, the constellation diagram can be partitioned into 2^K lines (where the total possible lines in the constellation diagram can be represented as $\int$) that can only intersect each other at the following points: $\aleph = \{\mathrm{T}b\}_{b \in \{-1,1\}^K} \setminus \int$. Figure 1 shows the constellation diagram that consists of three different vectors (lines) with the original vector '**X**' that represents the collective complexity of the receiver.

Q, R, and S represent vectors or transformation points within the coverage area of a cellular network (Figure 1). In addition, Q¬, R¬, and S¬ represent the computational complexity of each individual transformation point. In order to compute the collective computational complexity of the optimum receiver, it is essential to determine the complexity of each individual transformation point. The computational complexity of each individual transformation point is represented by X¬ of the transformation point which is equal to the collective complexity of Q¬, R¬, and S¬. A transformation does not change the original vector, instead it alters the components. In order to derive the value of the original vector **X**, we need to perform the following derivations. We consider the original vector with respect to each transmitted symbol or bit (i.e., the number of lines or vectors in the constellation diagram).

$$X^{\neg}Q = Xi^{\neg} = \left(XQ_i + XR_j + XS_K\right) i^{\neg}$$
$$= XQii^{\neg} + XRji^{\neg} + XSki^{\neg}$$
$$X^{\neg}R = Xj^{\neg} = \left(XQ_i + XR_j + XS_K\right) j^{\neg}$$
$$= XQij^{\neg} + XRjj^{\neg} + XSkj^{\neg}$$
$$X^{\neg}S = Xk^{\neg} = \left(XQ_i + XR_j + XS_K\right) k^{\neg}$$
$$= XQik^{\neg} + XRjk^{\neg} + XSkk^{\neg}$$

The following equation can be derived from the above system:

$$\begin{pmatrix} X^{\neg}Q \\ X^{\neg}R \\ X^{\neg}S \end{pmatrix} = \begin{pmatrix} ii^{\neg} & ji^{\neg} & ki^{\neg} \\ ij^{\neg} & jj^{\neg} & kj^{\neg} \\ ik^{\neg} & jk^{\neg} & kk^{\neg} \end{pmatrix} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \qquad (9)$$

Equation (9) represents the following: *QRS* with the unit vectors $i, j,$ and k, and $X^{\neg}Q$, $X^{\neg}R$, and $X^{\neg}S$ with the inverse of the unit vectors $i^{\neg}, j^{\neg},$ and $k^{\neg}$. The second matrix on the right hand side of (9) represents **b**, where as the first matrix on the right hand side of (9) represents the actual transformation matrix. Therefore, the transformation matrix from the global reference points (which could be *Q*, *R*, or *S*) to a particular local reference point can now be derived from (9):

$$\begin{pmatrix} X^{\neg}Q \\ X^{\neg}R \\ X^{\neg}S \end{pmatrix} = T_{L/G} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \qquad (10)$$

Equation (10) can also be written as:

$$T_{L/G} = \begin{pmatrix} ii^{\neg} & ji^{\neg} & ki^{\neg} \\ ij^{\neg} & jj^{\neg} & kj^{\neg} \\ ik^{\neg} & jk^{\neg} & kk^{\neg} \end{pmatrix} \qquad (11)$$

We need to compute the locations of the actual transformation

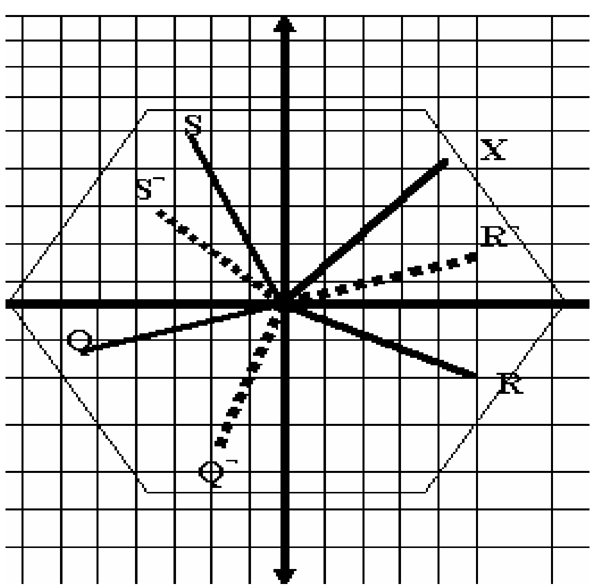

Figure1. A constellation diagram consisting of three different parameters

points described in (10) and (11). Let the unit vectors for the local reference point be:

$$\begin{aligned} i^{\neg} &= \left[t_{11}i, t_{12}\, j, t_{13}k \right] \\ j^{\neg} &= \left[t_{21}i, t_{22}\, j, t_{23}k \right] \\ k^{\neg} &= \left[t_{31}i, t_{32}\, j, t_{33}k \right] \end{aligned} \qquad (12)$$

since, $i^{\neg}\left(i + j + k\right) = i^{\neg}$, where $\left(i + j + k\right) = 1$. The same is true for the rest of the unit vectors. Therefore, (12) can be rewritten as:

$$\begin{aligned} i^{\neg} &= \left[t_{11}, t_{12}, t_{13} \right] \\ J^{\neg} &= \left[t_{21}, t_{22}, t_{23} \right] \\ k^{\neg} &= \left[t_{31}, t_{32}, t_{33} \right] \end{aligned} \qquad (13)$$

By substituting the values of $i^{\neg}, j^{\neg},$ and $k^{\neg}$ from (13) into (11), we obtain

$$T_{L/G} = \begin{pmatrix} i\left(t_{11}i+t_{12}j+t_{13}k\right) & j\left(t_{11}i+t_{12}j+t_{13}k\right) & k\left(t_{11}i+t_{12}j+t_{13}k\right) \\ i\left(t_{21}i+t_{22}j+t_{23}k\right) & j\left(t_{21}i+t_{22}j+t_{23}k\right) & k\left(t_{21}i+t_{22}j+t_{23}k\right) \\ i\left(t_{31}i+t_{32}j+t_{33}k\right) & j\left(t_{31}i+t_{32}j+t_{33}k\right) & k\left(t_{31}i+t_{32}j+t_{33}k\right) \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \qquad (14)$$

Substituting $\boldsymbol{T}_{L/G}$ from (14) into (10), yields

$$\begin{pmatrix} X^{\neg}Q \\ X^{\neg}R \\ X^{\neg}S \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \qquad (15)$$

Equation (15) corresponds to the following standard equation used for computing the computational complexity at the receiving end: $\aleph = \left\{\mathrm{Tb}\right\}\ \mathrm{b} \in \{-1, +1\}^{k}$. If the target of one transformation $(U : Q \rightarrow R)$ is the same as the source of other transformation $(T : R \rightarrow S)$, then we can combine two or more transformations and form the following composition: $TU : Q \rightarrow S,\ TU\left(Q\right) = T\left(U\left(Q\right)\right)$.

This composition can be used to derive the collective computational complexity at the receiving end using (15). Using (15), a simple matrix addition of the received demodulated bits can be used to approximate the number of most correlated transformation points. The entire procedure for computing the number of demodulated bits that need to be searched out by the decision algorithm can be used to approximate the number of most correlated signals for any given set of transformation points. This is because we need to check whether or not the transformation points are closest to either (+1, +1) or (-1, -1). The decision regions or the coordinates where the transformation points lie for (+1, +1) and (-1, -1) are simply the corresponding transformation matrices that store the patterns of their occurrences. If the transformation points do not exist in the region of either (+1, +1) or (-1, -1), then it is just a matter of checking whether the transformation points are closest to (+1, -1) or to (-1, +1). In other words, the second matrix on the right hand side of (15) requires a comprehensive search of at most 5^K demodulated bits that indirectly correspond to one or more users. The minimum search performed by the decision algorithm is conducted if the transformation points exist within the incorrect region. Since the minimum search saves computation by one degree, the decision algorithm has to search at least 4^k demodulated bits. The average number of computations required by a system on any given set always exists between the maximum and the minimum number of computations performed in each operational cycle [16]. This implies that the total number of demodulated bits that need to be searched out by the decision algorithm can not exceed by 5^K-4^K. Thus, the total number of most correlated pairs has an upper bound of 5^K-4^K.

Since most of the decisions are correct, we can reduce the number of computations by using the transformation matrices only on those coordinates that are most likely to lead to an incorrect decision. Therefore, this greatly reduces the unnecessary processing required to make a decision about the correct region. Thus, the number of received demodulated bits that need to be searched out can be approximated as 5^K-4^K.

The computational complexity of any multiuser receiver can be quantified by its time complexity per bit [12]. The collective computational complexity of the proposed algorithm is achieved after performing the transformation matrix sum using the complex properties of the existing inverse matrix algorithms. This implies that both quantities **T** and **b** from our fundamental equation can be computed together and the generation of all the values of the demodulated received bits **b** can be done through the sum of the actual transformation matrix **T** that approximately takes $\boldsymbol{O}\,(5/4)^k$ operations with an asymptotic constant. Using the Newton approximation method given in MATLAB, we can directly come to an approximation of $\boldsymbol{O}\,(5/4)^k$. The computational complexity of the proposed algorithm is not polynomial in the number of users, instead the number of operations required to maximize the demodulation of the transmitted bits and to choose an optimal value of **b** is $\boldsymbol{O}\,(5/4)^k$, and therefore the time complexity per bit is $\boldsymbol{O}\,(5/4)^k$. Even though the computational complexity of the proposed algorithm is not polynomial in terms of the total number of users, it still gives significantly reduced computational complexity and provides much better performance in terms of SNR and the BER than the other well-known multiuser detection algorithms.

3.3. Computing SNR based on the reduced complexity

Consider the following points: (a) $\aleph$ is a computational complexity that belongs to a certain coverage area, (b) SNR (we represent SNR by γ) is uniformly distributed among all the active users' signals with respect to the computational complexity, and (c) a certain cellular coverage area has $\boldsymbol{K}$ users. Since SNR is uniformly distributed among all the users' signals at the receiving end, each user experiences an average of γ/K SNR. In order to achieve maximum positive values of SNR for most of the values of *K*, we propose that the inverse of the computational complexity should equal the difference between the inverse-normalization factor and the product of inverse-normalization factor and SNR with respect to the collective computational complexity of the system. This hypothesis leads us to the following equation:

$$\frac{K}{\aleph} = C^{-1} - C^{-1}\frac{\gamma}{\aleph} = \frac{1}{C}\left[1 - \frac{\gamma}{\aleph}\right] \tag{16}$$

where C in (16) represents the normalization factor, $K/\aleph$ is the inverse of the computational complexity, and $\gamma/\aleph$ represents the SNR with respect to the collective computational complexity. The main objective of (16) is to ensure that we get maximum positive values of SNR for most of the values of K. Using the complexity and the user-domain, we can make an argument that the inverse of an average SNR should be at least greater than zero. This argument guarantees that the system does not work with a non-positive value of SNR. In other words, the inverse of the average SNR should equal to the difference of the normalization factor and the inverse of the average computational complexity. The above equation can be written as:

$$\gamma = \aleph - CK \tag{17}$$

Equation (17) can also be used to compute the values of SNR in an ideal situation only if MAI does not affect the received signals by K-1 users. However, in a practical DS-CDMA system, this assumption can not be made. Therefore, we should consider that the variations in the network load for an AWGN channel introduces the presence of variance (we represent variance by σ^2) that represents MAI. The selection of variance is entirely dependent on the network load. In order to compute the values of SNR in decibels (dB), we need to change the linear quantity into decibels (dB) by multiplying it with the base-10 logarithmic function as well as with the variance. Since MAI is a multiplicative property of SNR, the resultant approximation of SNR in dB is always a product of the base-10 logarithmic function and the possible variance with respect to the number of users.

$$\gamma = 10\sigma^2 \log_{10}\left(\aleph - CK\right) \tag{18}$$

We use the precomputed values of variance, given in [6], in our simulation that represents MAI for a range of users. Furthermore, the normalization factor represents a varying quantity that can be used to approximate the different values of SNR with respect to the difference between average computational complexity and average SNR. It should be noted that (18) only gives approximate values that can be closed to the actual values of SNR depending on both the variance and the normalization factor.

3.4. Computing BER based on the reduce complexity

We modeled the cellular network as a LTI synchronous DS-CDMA system in which users utilize an AWGN multipath channel. Due to the AWGN channel and the linearity property, the different signal components do not experience deep fades. In other words, if the signal changes during the transition, the receiver receives the following signal:

$$\Re(t) = Ae^{-j\theta} + s(t) + \eta(t)$$

where A is an attenuation factor, θ is a phase shift, $s(t)$ is the desired signal, and $\eta(t)$ is the additive Gaussian noise. Due to LTI characteristics, the proposed algorithm is independent of the phase shift, which permits us to ignore it by simply setting the value of θ to zero. Therefore, the receiver receives the following signal:

$$\Re(t) = s(t) + \eta(t) + A$$

Since the attenuation factor A is uncorrelated with $\eta(t)$, we can use the value of SNR directly in the BER formula. Consider (19) that can be used to determine the BER in an AWGN channel for a system where the transmitted bits are modulated using the BPSK modulation technique.

$$\text{BER} = Q\left[1/\sqrt{1/SNR}\right] \tag{19}$$

It should be noted that the signals from all the users are synchronized with each other and the power of each user's signal is equal to the energy per bit with respect to time. Since the attenuation factor and the white noise are uncorrelated, the SNR can be directly placed in (19) as follows:

$$\text{BER} = Q\left[1/10(SNR) + \sigma^2\right]^{-1/2} \tag{20}$$

Where $Q(x)$ is the Gaussian Q function [7]. For simplicity, (20) can also be written as:

$$\text{BER} = Q\left[\sqrt{10SNR}/\sqrt{1+10\sigma^2 SNR}\right] \tag{21}$$

The second term in (20) represents the SNR degradation due to MAI. This term depends on the cross-correlation between the spreading code as well as the number of users. In other words, an increase in K causes an increase in the second term of (20) which causes a decrease in the overall BER performance. Furthermore, the possibility of variance in network load can not be ignored while calculating the BER performance, since our numerical calculations for BER are based on SNR which itself contains a small random amount of variance, as shown in (20). Since, in a synchronous system, the value of variance also depends on the value of the cross correlation, we assume a constant cross correlation value for the entire computation of BER.

4. Performance analysis of the proposed algorithm

This section analyzes the computational complexity, SNR, and the BER performance of the proposed algorithm and compares it to the ND and the ML algorithms. The system is modeled as a synchronous DS-CDMA system in a Gaussian channel.

4.1. Complexity analysis

The order of growth of a function is an important criterion for analyzing the complexity and efficiency of an algorithm. It gives a simple characterization of the algorithm's efficiency and also allows us to compare the relative performance of algorithms with given input sizes. The original computational complexity of the ML optimal receiver is $(2)^k$ [3]. The ND algorithm [8] reduced the complexity from $(2)^k$ to $(3/2)^k$ whereas the proposed algorithm reduced the complexity by $(5/4)^k$.

Figure 2 shows the computational complexities for a network that consists of 100 users. As we can see the proposed algorithm for a network of 100 users requires fewer computations as compared to the ML and the ND algorithms. In addition, the proposed algorithm greatly reduces the unnecessary computations involved in signal detection by storing the pattern of occurrence of the demodulated bits in the transformation matrix and uses it only on those decision regions which are most likely to lead to an incorrect decision. Also, from the subsequent sections, we see that these significant computational savings do not come at the expense of performance. It should be noted that the computational complexity curve for the proposed algorithm is growing in a linear order rather than in an exponential order. As the number of users increases in the system, the computational complexity differences among the three approaches will be obvious.

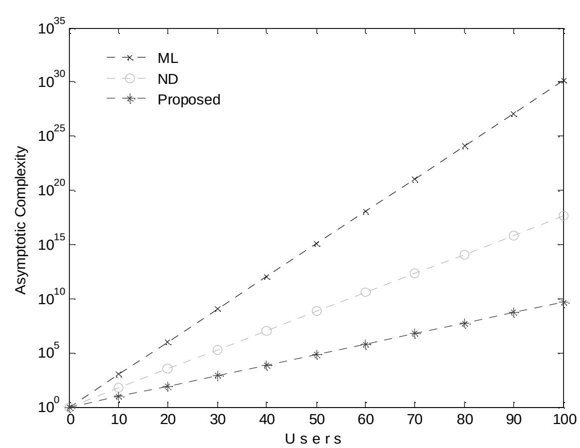

Figure 2. Computational complexities versus users

4.2. Performance analysis of SNR

Three detection algorithms are investigated, which are the original ML algorithm, ND algorithm, and the proposed algorithm. The values of SNR are approximated using (18) from Section 3.3. In order to compare the SNR performance of the proposed algorithm with the other multiuser detection algorithms, we use a same constant value with their computational complexities that does not make an exception for any one of the investigated algorithms. In our simulation, we use one (i.e., $C = 1$) as a normalization factor that remains same for all investigated algorithms. The choice of a small value of σ^2 is entirely based on the load of the coverage area (K) and it is selected through a random process for a certain range of users. For a lightly loaded network, we expect that the value of variance may vary from 0.6 to 0.9 and for a heavily-loaded network, the value of variance may vary from 0.1 to 1.

Figure 3 shows a network where 22 users are active within the coverage area of a cellular network. For a small value of K such as 2 or 4, the proposed algorithm offers approximately 6.5 dB of SNR where as the ND and the ML algorithms give 5.8 and 5.5 dB respectively. A slight increase in the value of K forces the proposed algorithm to give an acceptable value of the SNR that can be used to achieve a satisfactory BER performance for a voice communication network. For a small network, the three algorithms behave almost exactly the same. When the system becomes heavily loaded such as K = 42, the divergence rate for the proposed algorithm increases. This can be seen in figure 4, where the proposed algorithm has more rapid divergence with respect to the number of users when compared with the ND and the ML algorithms. The divergence for SNR is an essential element in achieving a minimum amount of BER. In other words, the divergence in SNR is directly proportional to the convergence in BER performance. In addition, it can be clearly observed from figure 4 that the linear increase in SNR for the proposed algorithm is smoother and more uniform than the ND and the algorithm is always better than the ML and the ND algorithms as shown in figure 5. For the first few values of SNR, the ND algorithm almost approaches the ML algorithm whereas the proposed algorithm still maintains a reasonable performance difference. It can be seen in figure 5 that the proposed algorithm achieves less than 10^{-2} BER for SNR = 8 dB which is quite closed to the required reasonable BER performance for a voice communication system. For small values of SNR, the BER for these three algorithms is almost equal, but as we increase the value of SNR, typically more than 10 dB, one can clearly observe the difference in the BER performance.

The BER curve in figure 6 is calculated using (21). The former result demonstrates a slight improvement over the BER performance shown in figure 5 for all SNR values above 9 dB. Even for small values of SNR, the proposed algorithm gives better performance than the ML and the ND algorithms. As the value of SNR increases, the BER performance of the proposed algorithm over the ND and the ML algorithms

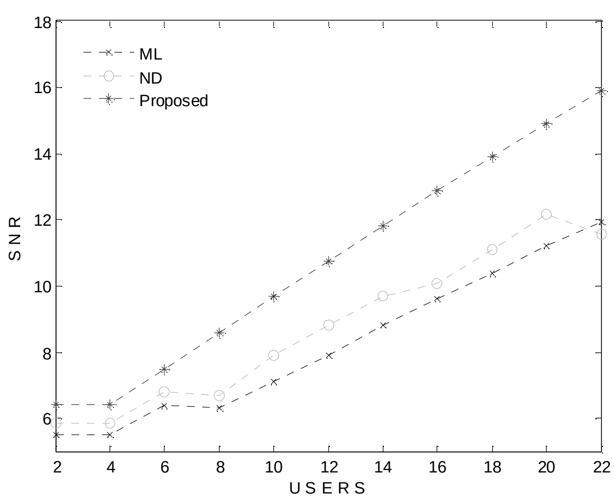

Figure 3. Approximate value of SNR (dB) versus number of users (K =22) with a random amount of variance for a synchronous DS-CDMA system in a Gaussian channel

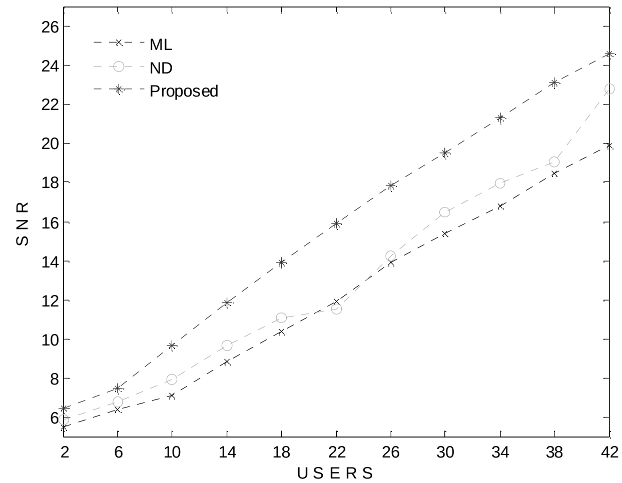

Figure 4. Approximate value of SNR (dB) versus number of users (K =42) with a random amount of variance for a synchronous DS-CDMA system in a Gaussian channel

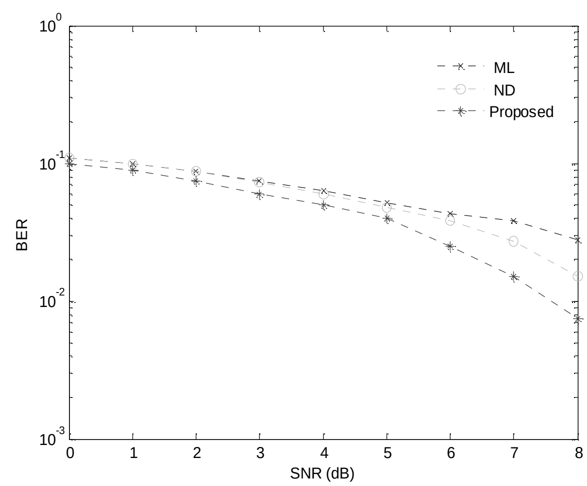

Figure 5. BER versus SNR (0<dB<9) curves

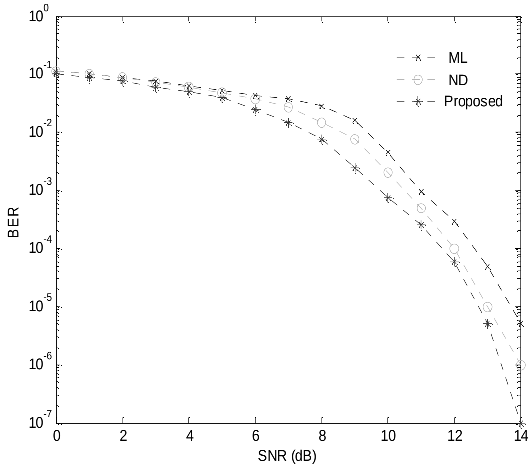

Figure 6. BER versus SNR (0<dB<14) curves

which further maintains the overall BER performance of multiuser receiver.

5. Conclusion

In this paper, a novel approach for reducing the computational complexity of multiuser receivers is proposed, which utilizes the transformation matrix technique to improve the performance of multiuser detectors. We present a mathematical model which verifies the implementation of the transformation matrix technique. The numerical results for the computational complexity of the proposed algorithm demonstrate the success of the proposed algorithm over the MD and the ND algorithms. Furthermore, we present a new mathematical model for computing the values of SNR. The main advantage of the proposed mathematical model for SNR is that it guarantees that the receiver does not process the signals that have non-positive values of SNR. In order to show the consistency and the correctness of the proposed approach, we present comprehensive simulation results for computing SNR with different ranges of users. The simulation results for SNR demonstrate the consistency of the desired values required to achieve an optimal BER performance. In addition, we present BER results not only for a lightly-loaded network but also for a heavily-loaded network. The BER simulation results of the proposed algorithm suggest that the proposed algorithm achieves better BER performance for all values of SNR than the other well-known multiuser detection algorithms.

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